The International Journal of Humanities

RESEARCH ARTICLE | SPECIAL ISSUE: PHILOSOPHY

Received: 2 October 2019
Accepted: 9 May 2020
Published: 1 January 2021

History of Contraposition in Ancient Logic

Asadollah Fallahi[1]

[1] Associate Professor, Logic Department, Iranian Institute of Philosophy,Tehran, Iran.
E-mail: falahiy@yahoo.com

How to cite this article
Fallahi, Asadollah. (2021). History of Contraposition in Ancient Logic, *The International Journal of Humanities* (2021) Vol. 28 (1): (69-80).

http://eijh.modares.ac.ir/article-27-39431-en.html

Abstract: The rule of contraposition was used by Aristotle and later popularized during the medieval period in both Arabic and Latin logics. In this paper, we investigate the role of Aristotle and ancient commentators on developing the subject. We show that although Aristotle had used contraposition (on conditionals and indefinite affirmative categorical propositions), Proclus was the first to apply it to *universal* affirmatives and Philoponus was the first to name the rule. The latter used the rule on *possible* propositions too, which yielded to Simplicius' thorough objections. We have found no ancient logician who could apply the rule to quantified categorical propositions except universal affirmatives, nor have we encountered any ancient logician who thoroughly investigated the rule with regard to all kinds of modal propositions. It seems that these developments occurred in later stages of the history of logic.

Keywords: Contraposition; Aristotle; Proclus; Philoponus; Simplicius

Introduction

The logical rule contraposition is ancient, having been used by Aristotle. It has pleasant history both in ancient Greek and in Medieval Latin and Arabic. The history of contraposition, as it seems, can be divided at least into two main parts: before and after Avicenna (980-1037). Before him, the most obvious feature of the rule of contraposition is that it has had been applied just to three kinds of propositions: conditionals; affirmative universal and/or indefinite categorical propositions and; possible indefinite categorical propositions. All these will be discussed in this paper.

The roles of Fārābī and Avicenna's innovations in regard to contraposition have been investigated (Fallahi, 2018), and this paper attempts to study the history of the rule in ancient times. We begin with Aristotle, the First Teacher.

Aristotle

Without naming 'contraposition', Aristotle used it both for conditionals (*An. Pr.* 53b12, 57a, 36-b17, and *Topics* 163a 32-36) and for affirmative indefinite categorical propositions (*Topics* 113b, 15-26). Examining Aristotle's texts, we shall see that the application of contraposition to both conditionals and categorical propositions are not as different as they may seem at first.

Aristotle on Conditional Contraposition

All applications of contraposition to conditionals we found in Aristotle's logical books are related to entailments or consequences in the form of syllogisms: if the conclusion is false then the conjunction of the premises is false. Here is the first text:

> First then that it is not possible to draw a false conclusion from true premises, is made clear by this consideration. *If it is necessary that B should be when A is, it is necessary that A should not be when B is not*. (Italics are ours.)[1]

Here we assume that the variables *A* and *B* range over propositions not on terms and that expressions like '*A* is' mean '*A* is true.' This is clear from the first sentence of the above text, which concerns with premises and conclusions. The same notes hold for the second text:

> It is clear then that *if the conclusion is false, the premises of the argument must be false*, either all or some of them; but when the conclusion is true, it is not necessary that the premises should be true, either one or all, yet it is possible, though no part of the deduction is true, that the conclusion may none the less be true; but not necessarily. The reason is that *when two things are so related to one another, that if the one is, the other necessarily is, then if the latter is not, the former will not be either*, but if the latter is, it is not necessary that the former should be. (Italics are ours.)[2]

The third text can be found in *Topics*:

> For *conversion* is taking the reverse of the conclusion together with the remaining propositions asked and so demolishing one of those that were conceded; for it follows necessarily that *if the conclusion is untrue, some one of the propositions is demolished*, seeing that, given all of them, the conclusion was bound to follow.(Italics are ours.)[3]

This text seems to name contraposition (or better, antilogism (if $p, q \vdash r$, then $p, \sim r \vdash \sim q$)) by the mere term 'conversion' and uses the word 'reverse' to mean contradictory opposition.

As it is well-known, Aristotle used to declare his syllogisms in the form of conditionals. So we can neglect the delicate difference between consequence and conditional and thus it is safe to regard Aristotle's applications of the rule as to conditionals indeed.

Aristotle on Categorical Contraposition

For the contraposition of categorical propositions, we also consider his *Topics*,

[1] Aristotle, *An. Pr.,* 53b11-53b25, Aristotle *1984*, 57, translation by Jenkinson.

[2] Aristotle, *An. Pr.,* 57a36-b17, Aristotle *1984*, 64, translation by Jenkinson.

[3] Aristotle, *Topics*, 163a32-36, Aristotle *1984b*, 135, translation by Pickard-Cambridge.

where he introduces some quasi-definitions with three examples:

> § 8 · Seeing that the modes of opposition are four in number [contradiction, contrariety, relation, and privation], you should *look among the contradictories of your terms, reversing the order of their sequence*, both when demolishing and when establishing a view; and you should grasp this by means of induction, e.g.
>
> [1] if 'man is an animal', 'what is not an animal is not a man'; and likewise also in other instances of contradictories. For here the sequence is reversed; for animal follows upon man, but not-animal does not follow upon not-man, but the reverse—not-man upon not-animal.
>
> In all cases, therefore, a claim of this sort should be made, (e.g.) that
>
> [2] if 'the honourable is pleasant', 'what is not pleasant is not honourable', while if the latter is not so, neither is the former. Likewise, also,
>
> [3]if 'what is not pleasant is not honourable', then 'what is honourable is pleasant'.
>
> Clearly, then, *reversing the sequence in the case of contradictories* is a method convertible for both purposes (Paragraphing, numbering, italics and the quotation marks are ours.)[4]

The italic parts of the text can be taken as quasi-defining what is now named 'contraposition.' They can be abbreviated as 'reversing the sequence of the contradictories of the terms.' This definition does not explicitly require the condition 'quality-preserving.' The importance of this condition is that later, Avicenna and many Arabic logicians have implicitly or explicitly violated it.

The above text contains three examples (identified by numbers in brackets). The first example shows an entailment from a proposition to its contrapositive, without indicating the converse entailment. The second and the third examples, however, show that this rule is reversible and two-sided (i.e. equivalence).

Aristotle's examples are affirmative indefinites. But his first example, 'man is an animal,' is clearly universally true and so can be interpreted as affirmative universal; but this universality is not so clear for the other two examples: 'the honorable is pleasant' and its contrapositive. The question here is how to interpret the indefinite examples in the text, as universals or as particulars? This is related directly to similar questions in some commentators, who will be discussed below. If contrary to what Aristotle commonly declares, we interpret all the indefinite examples as universal, he will be committed only to A-contraposition, which is, as we will see soon, what Proclus [and Fārābī] explicitly would express. If we interpret them as particular, as Aristotle commonly used to do, he will be committed to I-contraposition, which is exactly what Philoponus and Avicenna would, respectively, implicitly and explicitly commit themselves to.

Actually, we find a case for the universal interpretation, i.e. the verb 'follow upon' in the sequel of the first example: 'for animal follows

[4] Aristotle, *Topics*, 113b15-113b26, Aristotle *1984*, 29-30, translation by Pickard-Cambridge.

upon man, but not-animal does not follow upon not-man, but the reverse—not-man upon not-animal.' This verb shows that Aristotle interprets the first example as an implication or a conditional. But thanks to symbolic logic, we know that only universal categorical propositions can be analyzed to implications or conditionals (particular ones are analyzed to conjunctions). The next expression, 'in all cases, therefore, a claim of this sort should be made, (e.g.) that …,'seems to show that the other two examples should be interpreted as implications, conditionals, i.e. simply as universals.

The last note can be seen as a partial evidence that Aristotle dose not see a big hole between conditionals and (universal) predicative or categorical propositions and hence no such gap between contraposition of both kinds. As we see in the sequel, such gap would not be noticed in ancient logic. But the history of contraposition in Medieval Arabic logic will open a grate gap between the two kinds of propositions in regard to contraposition, which we consider in a next paper.

Alexander of Aphrodisias

We consider in Alexander of Aphrodisias (fl. 200) only two texts he seem to deal with contraposition: *in An. Pr.* 29, 16-18, and 46, 2-8. In the first, he repeated only Aristotle's first example of the last text mentioned above, which converts by contradiction an indefinite affirmative proposition:

> There is conversion together with an opposition among propositions too. For the proposition saying 'What is not an animal is not a man' converts from 'Man is an animal'.[5]

The second text is relevant in name but not in meaning:

> When propositions share their two terms with one another but the terms in them are not in the same order but are taken inversely - it is among propositions which share in this way that propositional conversions are found. For the conversion of propositions is a matter of their sharing their two terms, inversely posited, and in addition being true together. When they differ in quality, such propositional conversions require an opposition - and they are called '*conversions with opposition*'.[6]

Here, it seems that Alexander speaks about simple conversion with changing quality, which causes some inconsistency or opposition with the converted proposition. For example, the universal propositions: 'All A is B' and 'No B is A', which are 'conversions with opposition' in Alexander's meaning. It is not clear if Alexander believed that the particular forms, 'Some A is B' and 'Some B is not A', are inconsistent as well, although we may assume that no inconsistency here arises. But all this is no relevant to contraposition. The name '*conversions with opposition*,' however, may be the source of the same name by which Philoponus later coined contraposition (see below).

In a note to the first text above, Barnes refers to three or four pages on contraposition in Greek: 'Alexander *in Top.* 190, 26-193,7 (*on Aristotle, Top.* 113b15-26)', which

[5] Barnes *1991*, 83.

[6] Barnes *1991*, 106, 46, 2-8.

unfortunately, as we found out, has not yet been translated to English.

Proclus Lycius

Arguing for the eternity of the world, Proclus (412-485) used contraposition for *universal affirmatives*.

> And if it is imperishable, it is un-generated; 'since for everything that has come to be, there is a passing out of existence', says Socrates on the eve of *Timaeus*' discourse. … So if this is true, anything for which there is no passing out of existence is un-generated.[7]

The argument goes straightforwardly as below:

For *everything* that has come to be, there is a passing out of existence;

∴ If it is imperishable, it is un-generated;
∴ *Anything* for which there is no passing out of existence, is un-generated.[8]

Here, the slide between conditional and affirmative universal is much clear. As Philoponus (490-570) reports, this argument is originally from Porphyry (c.234 - c. 305):

> It is with this intention, then, that *Proclus* has composed the sixth of his proofs [for the eternity of the world], or rather in it too has once more transcribed the words of *Porphyry* for our benefit; for in his commentary on the *Timaeus* the latter quite clearly uses this same proof with a view to establishing that *Plato* too holds that the world is everlasting. For, assuming that the world is in *Plato*'s view imperishable, he concludes that it must also be un-generated; for if, as *Plato* himself says in the *Phaedrus*, <u>for anything that has come to be there is of necessity a subsequent passing out of existence</u>, it no doubt in every case follows by *conversion by negation* that <u>if a thing does not perish it has not come to be</u>. So if *Plato* clearly states that the world is imperishable, it is no doubt absolutely clear that it is also un-generated. (Italics and underlines are ours).[9]

On this translation, the premise is an affirmative universal and the conclusion a conditional with a particular categorical antecedent:

For *anything* that has come to be, there is a passing out of existence

---

∴ *If a thing* does not perish it has not come to be.

The same note can be seen in other translations. For instance, Lang and Macro *2001*, 63 quote a conditional with a particular categorical antecedent:

There is corruptibility for *every thing* that has been generated

---

∴ If it is truly the case that *a thing* has no corruptibility then it is un-generated.

But thanks to modern logic, we know that a conditional with particular antecedent is equivalent to a universal categorical one: i.e. the form $(\exists x\, Fx \to P)$ is equivalent to the form $\forall x\,(Fx \to P)$. Especially, if, as in the example, a

[7] Share *2005*, 13.

[8] As will be seen, in the various translations, many different words and expressions have been suggested for the subject and the predicate of the example above, such as 'come to be,' 'be generated,' 'passing out,' 'be corruptible.' In each case, we follow the very terms presented in the translations.

[9] Share *2005*, 17.

pronoun in the consequent refers to the particular quantifier in the antecedent, the two sentences will be synonyms; this is because the form ($\exists x\ Fx \rightarrow Gx$) will not be a wff and must be rephrased as $\forall x\ (Fx \rightarrow Gx)$.

Thus, the important point here is the shift from indefinite propositions to explicitly *universal* ones, which is new in the history of the subject.

Philoponus

The first names for contraposition

It seems Philoponus (490-570) was the first who, as we found, used for contraposition some names, or better, phrases: '*sun antithesei antistrophên*'[10] and '*kata tên sun antithesei antistrophên*,'[11] meaning: 'conversion with/by contradiction / negation / opposition / *antithesis*.'[12]

The note 36 in Share 2005 p. 131 reports some English translations for the Greek expressions in the contemporary logical books:

> 'Conversion by negation' is the term preferred by e.g. Joseph (*An Introduction to Logic* (Oxford, 1906), 215). A more literal translation would be 'conversion with opposition', or perhaps 'conversion [with substitution of] the contradictory'. M. J. Edwards (*Philoponus: On Aristotle Physics 3* (London & Ithaca, NY, 1994), 'Greek-English Index' under *antistrophê*) prefers 'inversion with negation'.[13]

The first deal with Modal contraposition

Philoponus also has used contraposition for other arguments than Porphyry and Proclus had done. In one case, the propositions involved are modal, especially possible ones. They occur in a second argument among many ones he offered to possibility of heavens being constituted of the four known simple elements: earth, water, air and fire. Because Philoponus' texts have been lost, we quote his second argument from his contemporary critic, Simplicius (490-560):

> [1] Secondly, if it is true that bodies which are different in nature (e.g. water and earth) *can* move with the same movement, then, by *contraposition*, it should also be true that bodies which move with different movements *can* be of the same nature, i.e. it is *not impossible* that the heavens are of the same nature as the sublunary bodies although they move with a different movement.[14]
>
> [2] If <bodies> that are different in nature like earth and water *can* move with the same movement, <then,>*converting with negation* (*sun antithesei antistrephon*), he [i.e. Philoponus] says, you will say: *there is nothing to prevent*<bodies> which move with a different and not the same movement from being of the same nature.[15]
>
> [3] If it is *possible* for things of a different nature, such as earth and water, to have the same motion, then, '*converting with antithesis*' (*as he says*), you will say: *nothing prevents* different things which do not have

---

[10] *σὺν ἀντιθέσει ἀντιστρεφήν*

[11] *κατὰ τὴν σὺν ἀντιθέσει ἀντιστρεφήν*

[12] See Wildberg 1987, 44 and 96, Share 2005, 131, and Mueller 2011, 61.

[13] Share 2005 p. 131.

For Porphyry, the next note 37 in Share 2005 p. 131 refers to Sodano 1964, 2, 39 and to 126, 10-23.

[14] Wildberg *1987*, 41.

[15] Wildberg *1987*, 44.

> the same motion from being of the same nature, so that, even if heaven moves in a circle, but sublunary things move in a straight line, *nothing prevents* heaven from being of the same nature as sublunary things and perishable like them.[16]

In the third text, Simplicius adds the phrase 'as he says', which may be an evidence that Philoponus was actually the first who coined the name (probably borrowing from Alexander's '*conversions with opposition*').

Analyzing Philoponus' Argument

As we see, the propositions in Philoponus' argument are modal: the phrases 'can move' and 'nothing to prevent' signify possibility. So the rule used here can be formalized as below:

$$\frac{A \text{ is possibly } B}{\therefore \text{ non-}B \text{ is possibly non-}A}$$

The premise and the conclusion of this rule can be read as *de re* or *de dicto*, and also, as particular or universal, providing the following four interpretations:

de re particular	(1)	$\dfrac{\exists x\,(Ax \,\&\, \Diamond Bx)}{\therefore \exists x\,(\sim Bx \,\&\, \Diamond \sim Ax)}$
de dicto particular	(2)	$\dfrac{\Diamond \exists x\,(Ax \,\&\, Bx)}{\therefore \Diamond \exists x\,(\sim Bx \,\&\, \sim Ax)}$
de re universal	(3)	$\dfrac{\forall x\,(Ax \to \Diamond Bx)}{\therefore \forall x\,(\sim Bx \to \Diamond \sim Ax)}$
de dicto universal	(4)	$\dfrac{\Diamond \forall x\,(Ax \to Bx)}{\therefore \Diamond \forall x\,(\sim Bx \to \sim Ax)}$

or simply in propositional logic:

conjunctive with narrow scope $\Diamond$	(1′)	$\dfrac{(A \,\&\, \Diamond B)}{\therefore (\sim B \,\&\, \Diamond \sim A)}$
conjunctive with wide scope $\Diamond$	(2′)	$\dfrac{\Diamond (A \,\&\, B)}{\therefore \Diamond (\sim B \,\&\, \sim A)}$
conditional with narrow scope $\Diamond$	(3′)	$\dfrac{(A \to \Diamond B)}{\therefore (\sim B \to \Diamond \sim A)}$
conditional with wide scope $\Diamond$	(4′)	$\dfrac{\Diamond (A \to B)}{\therefore \Diamond (\sim B \to \sim A)}$

Even in the very strong modal logic S5, the valid forms are just(4) and (4′).(The forms (1), (2), (1′), and (2′) are invalid also even in the strongest consistent modal logic Triv).[17]

Jan Mueller has formalized Philoponus' argument as (2′), conjunctive with wide scope possibility: 'If POS ($N(x,y)$ & ¬ $M(x, y)$) then POS ($M(x, y)$ & ¬ $N(x, y)$)',[18] which is invalid, as we said, even in the strong modal logics S5 and Triv. However, if this interpretation is true, the shift form affirmative universals to affirmative particulars in the application of contraposition can be regarded as a novelty in Philoponus.

We agree with Mueller in his formalization and try to somehow justify it; because of the examples Philoponus presented (water and earth), it seems to us that he intended the form

[16] Mueller *2011*, 61.

[17] For invalidity of (3) in S5 take a model with two worlds *w* and *w′* and a singleton {*a*} as their domains. Let *A* be true of *a* in both *w* and *w′* and *B* true only in *w′*. Then, in *w*, the premise of (3) will be true and its conclusion false. For invalidity of (1) and (2) in Triv take a model with a single world *w* and a singleton {*a*} as its domain. Let *A* and *B* be true of *a* in *w*. Then, in *w*, the premises of (1) and (2) will be true and their conclusions false.

[18] Mueller *2011*, 17.

(2), the *de dicto* particular interpretation (similar to Mueller's analysis (2′)):

$$(2)\quad \frac{\Diamond\exists x\,(Ax\&Bx)}{\therefore \Diamond\exists x\,(\sim Bx\&\sim Ax)}$$

de dicto particular interpretation

Invalidating Philoponus' Argument by Aristotelian Tools

We provided a counter-model for this contraposition in a footnote. For its invalidity in a more or less Aristotelian paradigm, it suffices for us to take '*C*' as an abbreviation of '~ *B*', (meaning 'difference in movement'). Thus, (2) will be rephrased as:

$$(2\text{-}1)\quad \frac{\Diamond\exists x\,(Ax\,\&\sim Cx)}{\therefore \Diamond\exists x\,(Cx\,\&\sim Ax)}$$

de dicto particular interpretation

which is equivalent to:

$$(2\text{-}2)\quad \frac{\sim\Box\forall x\,(Ax\rightarrow Cx)}{\therefore\ \sim\Box\forall x\,(Cx\rightarrow Ax)}$$

equivalent *de dicto* interpretation

This last formal argument would say that because difference in nature does not entail difference in movement, then difference in movement does not entail difference in nature. This, in turn, is equivalent to the statement that because difference in movement entails difference in nature,then difference in nature entails difference in movement. I. e. negating the conclusion of (2-2) yields the negation of its premise, so we reach the following third equivalent version of (2):

$$(2\text{-}3)\quad \frac{\Box\forall x\,(Cx\rightarrow Ax)}{\therefore \Box\forall x\,(Ax\rightarrow Cx)}$$

equivalent *de dicto* interpretation

But (2-3) is the direct conversion of entailment to entailment (or, of necessary A-proposition to necessary A-proposition), which is evidently invalid in Aristotle (because A-propositions are converted not to A- but to I-propositions, as Aristotle had taught). So we showed by some easy transformations that (2) is equivalent to (2-3), an invalid immediate argument in Aristotelian logic. If our argument is correct, it shows that Philoponus has deviated from Aristotle in a significant way.

Simplicius

Philoponus' example of contraposition has been announced as invalid by his contemporary Simplicius (490-560) and five centuries later by the Persian logician Avicenna (970-1037). We quote Simplicius:

Philoponus' second argument is *formally invalid*.[19]

Are Philoponus and Simplicius discussing the same rule, so that one of them is right, and the other wrong? Or are they considering two different interpretations of the same locutions? Simplicius has not given in this text any clue why he denied Philoponus' second argument. But, in the sequel of the same discussion in his other book *On Aristotle on the heavens*, Simplicius, as I read him, raised against the Philoponus' argument four objections.[20] Below I recover for these objections.

Simplicius' Objections

1. Adding negation after modality (not before that) is contrary to the law of making contradictories;

[19] Wildberg *1987*, 41.

[20] Mueller *2011*, 61-3.

2. Possibility-contraposition is invalid, even if the negation is added before modality,
3. Premise of the argument is false,
4. Conclusion of the argument (i.e. the claimed contrapositive) is false.

Then Simplicius ridicules Philoponus as being unaware of the principles of logic. After briefly discussing their disputation, Jan Mueller gives the right to the former to ridicule the latter; but then he counts their dispute as non-clear, and its logical analysis as unfruitful:

> But [Simplicius'] discussion is not very clear or precise, and I think we have no reason to suppose that Philoponus was any more clear or precise. I doubt that any attempt to make sense of the dispute in logical terms would be fruitful. (*Ibid.* 17).

We think, however, that their quarrel is not as unanalyzable as Mueller thinks. For our purposes, Simplicius' first two objections are more important, so we discuss them.

Analyzing Simplicius' First Objection

Simplicius' first objection deals with the scope of negation in the contraposition, which is narrow in Philoponus' argument, but which should be wide in Simplicius' view:

> However, first of all, he does not add the negating particle to the modal operator in the assumption of the contradictory denial of the affirmative consequent, as the dialectical rule requires. For he says '*it is possible ... to have the same motion*' and, wanting to take the contradictory denial of this affirmation, he does not say '*it is not possible … to have the same motion*' as he ought to say in adding the negating particle to the modal operator; rather he says, '*It is possible that things which do not have the same motion do not have different natures*'. So how is it possible for a person who does not know the denial which is contradictory of the consequent of the affirmation of the antecedent to understand conversion with antithesis? And what do I mean by conversion with antithesis? How is it possible for the person who does not know how denials are produced from affirmations to understand any kind of syllogism whatsoever? (*Ibid.* p. 61 (28, 18 – 28)).

The italicized sentences are two proposed contrapositives. So, Philoponus' argument goes as below:

> It is possible for things different in nature to have the same motion
>
> ---
>
> ∴ It is *possible* that things which *do not* have the same motion
> *do not* have different natures (*Ibid.* p. 61 (28, 23-4)).

But in Simplicius' view, it should be as follows:

> It is possible for things different in nature to have the same motion
>
> ---
>
> ∴ [If] it *is not possible* for things to have the same motion
> [they *do not* have different natures](*Ibid.* p. 61 (28, 21-2)).

It seems for us that this objection would be effective if possibility in the premise were in the consequent of the premise; but since Philoponus' argument has possibility in the beginning of the premise there is no justification to put the negation over the possibility and put the whole in the antecedent of a conditional, as Simplicius has done. In Philoponus' section above, we preferred the *de* *re* particular interpretation for his argument, i. e. form (2). This interpretation is invalid (as we saw) but does not confuse wide-narrow scope of negation as Simplicius thought.

Analyzing Simplicius' Second Objection

It yields to another conclusion: if our interpretation of Philoponus is true, then although he is wrong in his use of this invalid form, but this invalidity is not what Simplicius has claimed in his second objection.

Simplicius here claims that even if we put the negation before the modality, the obtained form is not valid (*Ibid.* 61-2 (28, 29 – 29,7)):

If it is *A*, it is possible that it is *B*

∴ If it is not possible for it to be *B*, it is not *A*.

He divides this form to two kinds (or matters):
(1) when the possibility of *B* holds of *all* the individuals of *A*,
(2) when it holds of *some* of what is *A*.

The form is valid, as Simplicius indicated, only for the first kind and has many counter examples for the second one. He presented only one example for the first kind and four for the second. His single example for the first (*Ibid.* p. 62 (29, 10-11)) is this:

If it is a human being it is possible that it is literate

∴ If it is not possible for it to be literate it is not a human being.

This is analyzed as below:

$$\frac{\forall x\,(Ax \rightarrow \Diamond Bx)}{\therefore \forall x\,(\sim \Diamond Bx \rightarrow \sim Ax)}$$

which is a substitute instance of a valid form in predicate logic. The premise of the example seems true and its conclusion is true too.

The four examples of the second kind (*Ibid.* pp. 62-3 (29, 15 – 30, 7)) are logical, mathematical, physical, and biological ones, as follows:

Logical: It is possible for things which are different in species to fall under the same genus

∴ Things which cannot fall under the same genus are not different in species.

Mathematical: If it is even, it is possible that it is not divisible <by two> down to the monad[21]

∴ If it is not possible that it is not divisible down to the monad then it is not even.

Physical: If it is different in nature it is possible to have the same motion

∴ If it is not possible for it to have the same motion then it is not different in nature.

Biological: If it is an animal it is possible for it to move its upper jaw

∴ If it is not possible for it to move its upper jaw it is not an animal.

As we see, all the premises and the conclusions (but the premise of the first logical example) are conditionals. Simplicius has claimed that in all the examples, the possibility of the consequents in the premise does not follow *all* the individuals of their antecedents, but only *some* of them; so the argument is invalid. Thus we cannot analyze these examples as below:

$$\frac{\forall x\,(Ax \rightarrow \Diamond Bx)}{\therefore \forall x\,(\sim \Diamond Bx \rightarrow \sim Ax)}$$

[21] i.e. 2^n, a power of 2

but as the following:

$$\frac{\exists x\,(Ax \rightarrow \Diamond Bx)}{\therefore \exists x\,(\sim \Diamond Bx \rightarrow \sim Ax)}$$

However, contrary to what Simplicus claimed, this form is valid. So we have to formalize the examples as the more correct form shown below:

$$\frac{\exists x\,(Ax \,\&\, \Diamond Bx)}{\therefore \exists x\,(\sim \Diamond Bx \,\&\, \sim Ax)}$$

and interpret Simplicus' verbs 'follow' as simple conjunction (!?). This is invalid of course, but is this what Philoponus had in mind? We doubt it and it seems that the latter's intention was what we previously indicated:

$$\frac{\Diamond \exists x\,(Ax \,\&\, Bx)}{\therefore \Diamond \exists x\,(\sim Bx \,\&\, \sim Ax)}$$

This is an invalid argument too, but not the one Simplicius meant. So, his claim of invalidity is correct but his reason for it seems wrong.

As we shall see in a next paper, Avicenna will object to Philoponus by declaring the invalidity of the possibility-contraposition in the form Avicenna accused Philoponus having used, i.e. possibility as modality not as part of the predicate. This objection will be new but will not deal with the heart of Philoponus' argument either.

Conclusion

The rule of contraposition has strong relations with other logical rules such as Modus Tollens, Reduction ad Absurdum, Antilogism (if p, $q \vdash r$, then p, $\sim r \vdash \sim q$), but it has not been thoroughly investigated in ancient age. As it may be clear, these rules are primarily in the realm of propositional logic; but Aristotle had used contraposition for categorical propositions. This, as we showed, led the rule to be applied to modal categorical propositions such as possible particular affirmatives.

This paper does not claim any inclusive research into the subject in ancient times; but this partial excavation shows that contraposition had not even a common known name except for near the end of the period. There might be in ancient age, some unknown investigation after Simplicius' objections on Philoponus' use of contraposition, which should be studied. The other option is to glance at Medieval Arabic, Hebrew and Latin works, which may mirror the last researches of ancient times.

References

[1] Aristotle, (1984). Prior Analytics, Translated by Arthur J. J. Jenkinson, in Barnes, Jonathan (ed.), *The Complete Works of Aristotle*, (The Revised Oxford Translation), Princeton University Press.

[2] Aristotle, (1984b). *Topics*, Translated by W. A. Pickard-Cambridge, in Barnes, Jonathan (ed.), *The Complete Works of Aristotle*, (The Revised Oxford Translation), Princeton University Press.

[3] Aristotle, (1989). *Prior Analytics*, translated by Robin Smith, Cambridge, Hackett Publishing Company.

[4] Barnes, J. *et al.*, (1991). *Alexander of Aphrodisias: On Aristotle Prior Analytics 1.1-7*, (London & Ithaca, NY).

[5] Fallahi, Asadollah, (2019). 'Fārābī and Avicenna on Contraposition', *Journal of History and Philosophy of Logic*, 40-1, pp. 22-41.

[6] Lang, H.S. and Macro, A.D. (2001). *Proclus On the Eternity of the World (De Aeternitate Mundi)*, translated by Lang, H.S. and A.D. Macro, (Berkeley, LA & London).

[7] Mueller, Ian, (2011). *Simplicius on Aristotle on the heavens 1.2-3*, Translated by Ian Mueller, London, Duckworth.

[8] Share, Michael, (2005). *Philoponus: Against Proclus On the Eternity of the World 6-8*, Translated by Michael Share, (London & Ithaca, NY).

[9] Sodano, A.R. (1964). (ed.), *Porphyrii in Platonis Timaeum Commentariorum fragmenta*, Naples.

[10] Wildberg, Christian, (1987). *Philoponus: Against Aristotleon the Eternity of the World*, Translated by Christian Wildberg, (Gerald Duckworth & Co. Ltd.). (Reprinted 2014).

The International Journal of Humanities

تاریخچهٔ عکس نقیض در منطق یونان

اسدالله فلاحی[1]

تاریخ دریافت: ۱۳۹۸/۷/۱۱
تاریخ پذیرش: ۱۳۹۹/۲/۲۰
تاریخ انتشار: ۱۳۹۹/۱۰/۱۲

[1] دانشیار مؤسسهٔ پژوهشی حکمت و فلسفهٔ ایران، تهران، ایران
E-mail: falahiy@yahoo.com

چکیده: قاعده عکس نقیض یکی از مهم‌ترین قواعد منطق سنتی است که هرچند ارسطو آن را در عمل به کار برده است، منطق‌دانان بعدی بودند که این قاعده را به عنوان یک قاعده منطقی بازشناختند و فراگیر ساختند. در این مقاله، نقش ارسطو و شارحان باستان در تطور این قاعده بررسی شده است. نشان داده‌ایم که هرچند ارسطو این قاعده را در مورد شرطی‌های متصل و گزاره‌های حملی موجب «مهمل» به کار برده است، پروکلس نخستین کسی است که عکس نقیض را به‌صراحت بر حملی‌های موجب «کلی» اعمال کرده و یحیای نحوی (فیلوپونوس) نخستین کسی است که برای این قاعده نامی قرار داده است. یحیی هم‌چنین این قاعده را بر گزاره‌های «ممکن» جاری ساخته و اعتراض‌های معاصر خود، سیمپلیکیوس، را برانگیخته است. با همهٔ اینها، هیچ منطق‌دانی در دوران باستان نیست (یا ما نمی‌شناسیم) که عکس نقیض گزاره‌های محصور به‌جز موجب کلی را به دست داده یا بررسی کرده باشد، چنانکه منطق‌دانی را در این دوره نیافته‌ایم که بحث مستوفایی از قاعده عکس نقیض در موجهات ارائه کرده باشد. به‌نظر می‌رسد که گسترش این قاعده به همهٔ محصورات و همهٔ موجهات در دوره‌های متاخرترِ تاریخ منطق رخ داده است.

واژه‌های کلیدی: عکس نقیض، ارسطو، پروکلس، یحیای نحوی، سیمپلیکیوس