J. Humanities (2003) Vol. 10 (1): 55-62

"Ferio"(EI-O) the Most Fundamental Mood in Aristotelian Categorical Logic

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Abstract

Aristotle in the *Organon* and Aristotelian Logicians in exposition of categorical logic have illusdrated the mechanism that provide proof of the figures Valid moods (figures 2,3,4) based on the four valid moods of the first figure syllogism, i.e *Barbara, Celarent, Darii* and *Ferio*. Aristotle subsequently simplified the categorical syllogism by showing that the moods *Barbara* and *Celarent* implied all the other moods.

In this article, the auther having taken recourse to one of the moods, i. e. *Ferio* (the weakest valid mood of the first figure) has assumed as axiomatic the system of categorical logic and proved all valid categorical argument forms based on *Ferio*.

Introduction

Aristotle in the ***Organon*** devised a system of categorical logic and established it based on valid moods of the syllogistic first figure i.e ***Barbara*** (AA-A), ***Celarent*** (EA-E), ***Darii*** (AI-I) and ***Ferio*** (EI-O) (*Aristotle, pri-Analy, 1949*). The Aristotelian logical system, in fact, was the first deductive (axiomatic) system in the history of science and this method that originated from his ***Organon*** was successfully applied to other scientific systems, for example in mathematics, physics and philosophy.

This is because one of the most important attributes in the deductive system is the independence of its axioms

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or preliminary rules, Now, the basic problem that raised about Aristotelian categorical logic is that, whether we could reduce axioms of this system to less than the four first figure moods or not?

Proving syllogistic moods

The method of proving valid moods of the second, third and fourth figure based on the valid moods of the first figure apparently have been introduced by Aristotle and Aristotelian logicians.

We know that in Latin the logical tradition for simplifying education of this system is that all valid moods have a name as follows. These names introduce logical structure and a method of proof for every valid mood.

(figure 1): ***Barbara*** (AA-A), ***Celarent*** (EA-E), ***Darii*** (AI-I), ***Ferio*** (EI-O)

(figure 2): ***Cesare*** (EA-E), ***Camestres*** (AE-E), ***Festino*** (EI-O), ***Baroco*** (AO-O)

(figure 3): ***Darapti*** (AA-I), ***Datisi*** (AI-I), ***Disamis*** (IA-I), ***Felapton*** (EA-O), ***Ferison*** (EI-O), ***Bocardo*** (OA-O)

(figure 4): ***Bramantip*** (AA-I), ***Camenes*** (AE-E), ***Fesapo*** (EA-O), ***Fresison*** (EI-O), ***Dimaris*** (IA-I) (***Kneale***, *1962, p. 232*)

It should be noted that Aristotle did not mention a fourth figure independently, because he had a special criterion for his division.

His basis of judgment was the width of the middle term in comparison with the minor and major terms and there are only three possibilities, it may be wider than one and narrower than the other (first figure), or wider than either (second figure), or narrower than either (third figure) (*Ross, 1964, p. 35*).

Logicians after Aristotle by designing a new criterion ie, position of the middle term, especially after Galen, gave authenticity to the fourth figure (*Nabavi, 1998b, p. 105*). There is no reference also in Aristotle's ***Organon*** to the subaltern syllogism (weakend moods), i.e. ***Barbara*** (AA-I), ***Celaront*** (EA-O), ***Cesaro*** (EA-O), ***Camestrop*** (AE-O) and ***Camenop*** (AE-O).They were introduced by the perpatetic Ariston of Alexandria in thst century B. C (*Lejewski, 1972, p. 516*).

The letters ***B, C, D*** and F in the beginning of valid mood names respectively refer to ***Barbara***, ***Celarent***, ***Darii*** and ***Ferio*** and means that in logically proving a mood we must use these first figure valid moods. In addition the letters ***s, p*** and ***m*** respectively refer to "simple conversion", "conversion by limitation" and "commutation of premises", the letter ***c*** in the middle of the name also refers to *reductio-ad-absurdum*. Later Aristotle discovered that two valid moods ***Barbara*** and ***Celarent***, are sufficient as axioms of categorical system.

"Lejewski. C. L" in his article Ancient Logic in the ***Encyclopedia of Philosophy*** Says:

Aristotle subsequently simplified the axiomatic foundations of his syllogistic method by showing that the syllogisms ***Barbara*** and ***Celarent*** implied all the other syllogisms (*Lejewski, 1972, p. 517*).

In the following, according to Aristotle's opinion we prove moods ***Darii*** and ***Ferio*** based on ***Celarent***.

(***Darii***) (AI-I) 1-B*a*C

2-A*i*B ∴ A*i*C

3-A*e*C contradictory of conclusion (assumption)

4-C*e*A simple conversion 3

5-B*e*A ***Celarent*** 4,1

6-A*e*B simple conversion 5, contradicts 2, (false)

7-A*i*C *Reductio-Ad-Absurdum* 3-6

(***Ferio***) (EI-O) 1-B*e*C

2-A*i*B ∴ AoC

3-A*a*C contradictory of conclusion (assumption)

4-CeB simple conversion 1

5-AeB ***Celarent*** 4,3, contradicts 2 (false)

6-AoC *Reductio-Ad-Absurdum* 3-5

In the two above mentioned proofs the following ordinary definitions have been used.

A= S*a*P =df all S is P (universal affirmative)

E= S*e*P =df no S is P (universal negative)

I= S*i*P =df some S is P (particular affirmative)

O= S*o*P =df some S is not P (particular negative)

Aristotle also found that we may take valid moods of whatever figure as axiomatic and prove another based on it (*Bochenski, 1968. p. 54*).

Organon and the nature of a negative proposition

One of the most important problems in Aristotelian categorical logic is the interpretation of negation in quantified propositions. Does negation refer to categorical relation (inclusion, copula) or does it refers to a predicate? We know that Aristotle repudiates any attempt to reduce the negation to the affirmation by saying that "A is not B" really means "A is not – B" (*Ross, 1964, p. 29*).

Based on Aristotle's opinion negation in "A is not B" refers to copula or inclusion but negation in "A is not – B" refers to the predicate. With more precise consideration Aristotle believes That "A is not – B" is narrower than "A is not B" and therefore does imply it, but he rejected the converse entailment which is required for the "obversion" rule (*Kneale, 1962, p. 57*). Aristotle says.

… whether … the expressions "not to be this" and "to be not - this" are identical or different in meaning … for they do not mean the same thing, nor is "to be not – white" the negation of "to be white" but "not to be white" *(Aristotle, Pri-Analy, 1949, 46 (51a, 5-10)*.

The proposition "no man is just" follows from the proposition "every man is not just" *(Aristotle, De – Int, 1949, 10 (20a, 20-24)*.

As we see Aristotle belived that the meaning of "is not-just" and "is not-white" is narrower than "is not just" and "is not white" and we know that a narrower concept logically does imply a broader one.

From a modern first-order predicate logic point of view, we could evaluate Aristotle's opinion "no man is just" must be formalized to $(\forall x){\sim}(Mx \supset Jx)$. And "every man is not-just" must be formalized to $(\forall x)(Mx \supset {\sim}Jx)$. In modern logic we know that, V. Vs Aristotle's opinion, $(\forall x)(Mx \supset {\sim}Jx)$ follows from $(\forall x){\sim}(Mx \supset Jx)$ by following proof based on natural deduction method (*c.f. Nabavi, 1998a*).

1- $(\forall x){\sim}(Mx \supset Jx)$

$\therefore (\forall x)(Mx \supset {\sim}Jx)$

2-Mx	*AP*
3- ${\sim}(Mx \supset Jx)$	*1,* $\forall E$
4- ${\sim}({\sim}Mx \vee Jx)$	*3, Impl*
5- ${\sim}{\sim}Mx \wedge {\sim}Jx$	*4, Dem*
6- ${\sim}Jx$	*5,* $\wedge E$
7- $Mx \supset {\sim}Jx)$	*2-6,* $\supset I$
8- $(\forall x)(Mx \supset {\sim}Jx)$	*7,* $\forall I$

But deducing $(\forall x){\sim}(Mx \supset Jx)$ from $(\forall x)(Mx \supset {\sim}Jx)$ is invalid, because if we let an interpretation (position) with one individual (object), this argument is reduced to the following argument.

$Ma \supset \sim Ja$

$\therefore \sim (Ma \supset Ja)$

By use of the "truth assignment method" we find the following counter interpretation that shows the invalidity of this argument.

$$\frac{Ma \quad Ja}{F \quad\;\; F}$$

Now, a more important problem could be raised, whether the grammatical and logical structure in a negative proposition are identical or not? If we accept Aristotle's opinion and we refer the negation in ordinary (natural) language to copula or the relation of inclusion, we arrive at a meaning V. VS ordinary semantics, because when we say "No Iranian is European" if we analyse this proposition to $(\forall x){\sim}(Ix \supset Ex)$ i.e., negation refers to copula, this formula is equivalent to $(\forall x)(Ix \wedge {\sim}Ex)$ i.e., every human being Iranian and not European and this is appearently false.

Logicans subsequent to Aristotle by accepting the logical law of obversion give a precise interpretation to negation in ordinary language. From this perspective, the negation does not refer to inclusion, categorical relation or copula but to these the predicate. Therefore based on the law of obversion "no A is B" and "every A is not – B", are identical premises and equivalent. In modern logic they formalized as $(\forall x)\ (Ax \supset {\sim}Bx)$.

"*Ferio*" and proving valid moods

As we noted, Aristotle finally established the categorical logic system based on two valid moods of the first figure i.e. ***Barbara*** and ***Celarent***. If we accept the law of Obversion, as do most Aristotelian (SP?) logicians in the history of logic, we can establishe this system based on one mood.

In this section, the author having utilized one of the moods i.e., ***Ferio***, has attempted to make axiomatic the system of categorical logic and prove all valid argument forms based on ***Ferio***. We show all quarter quantifieds A, E, I, O respectively "S*a*P", "S*e*P", "S*i*P" and "S*o*P" In the following table we introduce five basic rules of categorical logic, i.e.

1- ***Ferio*** (EI-O), 2- Obversion (Ob), 3- subalternation (SA), 4- Quantification Negation (QN), 5- (*Reductio-Ad-Absurdum*) (*RAA*)

<table>
<tr>
<td>Ferio (F):
MeP
<u>SiM</u>
∴ SoP</td>
<td colspan="2">Obversion (Ob)
$\dfrac{\therefore\ ...aP\ (...\ iP\ )}{\therefore\ ...eP\ '(...\ oP\ ')}$
$\dfrac{\therefore\ ...eP\ (...\ oP\ )}{\therefore\ ...eP\ '(...\ iP\ ')}$</td>
<td>Subalternation (SA)
$\dfrac{Sap\ \ (SeP\ \ )}{\therefore\ SiP\ \ (SoP\ \ )}$</td>
</tr>
<tr>
<td colspan="2">Quantification Negation (QN)
$\dfrac{\therefore\ not\left(\dfrac{SaP\ (SeP\ )}{SiP\ (SoP\ (}\right)}{\therefore\ \dfrac{SoP\ (SiP\ )}{SeP\ (SaP\ )}}$</td>
<td colspan="2">Reductio-Ad-Absurdum (RAA)
→ premises ($p_1, p_2, \dots, p_n$)
∴ C (conclusion)
not [C] AP
.
.
.
SoS (some s is not s)
∴ C RAA</td>
</tr>
</table>

Some points must be taken into consideration with regard to the above table.

1- P' is a complement of p and means not – p

2- Introduction to the rule of *reductio-ad-absurdum*

Firstly: we use the method of modern symbolic logic in the "rule of indirect proof" that proves contradictory in one phrase ($\phi \wedge \sim \phi$) but not in comparison phase two Secondly: the structure introduced for "RAA" in the table was familiar among Aristotelian commentators especially in Al-Farabi's book (*c.f. Rescher, 1964, p. 122)*. "Nasir Al Din Al Tousi" in ***Sharh Mantiq Al-Isharat wa-l-Tanbihat*** describes Al-Farabi's method and says:

"Al-Farabi illustrates a syllogism from proposition "some B is C" i.e., contradictory of conversion (or conclusion) and the proposition "no C is B", i.e. the original proposition that it's conversion be considered. The conclusion of this syllogism is "some B is not B" and this is absurdity while Avicenna admires Al-Farabi's proof" *(Al-Tousi, 1984, p. 199)*.

We can show Al-Farabi's proof in the following form.

1- no C is B	
∴ no B is C	
→ 2- some B is C	contradictory of conclusion (*assumption*)
3- some B is not B	***Ferio*** from 1 and 2, (*false*)
4- no B is C	*Reductio-Ad-Absurdum 2-3*

Extension and development of categorical logic

If we accept the "***Ferio***" as a fundamental mood in categorical logic, all coversion rules (simple conversion, conversion by limitation and conversion by contradiction and other valid moods especialy ***Barbara*** (AA-A), ***Celarent*** (EA-E) and ***Darii*** (AI-I), all valid syllogistic moods of figures 2, 3 and 4 are thus provable as follows.

(1): 1- A*a*B (A→I) (conversion by limitation)

∴ B*i*A	
→ 2- not [B*i*A]	AP
3- B*e*A	2 (QN)
4- *AiB*	1 (SA)
5- A*o*A	4, 3 (*F*)
6- B*i*A	2-5 (*RAA*)

(2): 1- A*e*B (E→E) (conversion)

∴ B*e*A	
→ 2- not [B*e*A]	AP
3- B*i*A	2 (QN)
4- B*o*B	3,1 (*F*)
5- B*e*A	2-4 (*RAA*)

(3): 1- A*i*B (I→I) (conversion)

∴ B*i*A	
→ 2- not [B*i*A]	AP
3- B*e*A	2 (QN)
4- A*o*A	1,3 (*F*)
5- B*i*A	2-4 (*RAA*)

(4): 1- A*a*B (A→A) (contraposition)

∴ B'*a*A'	
→ 2- not [B'*a*A']	AP
3- B'*o*A'	2 (QN)
4- B'*i*A	3 (Ob)
5- A*e*B'	1 (Ob)
6- B'*o*B'	4, 5 (*F*)
7- B'*a*A'	2-6 (*RAA*)

(5): 1- A*e*B (E → o) (contraposition by limitation)

∴ B'*o*A'

2- not [B'*o*A']	AP
3- B'*a*A'	2 (QN)
4- B'*e*A	3 (Ob)
5- A*o*B	1 (SA)
6- A*i*B'	5 (Ob)
7- A*o*A	6, 4 (*F*)
8- B'*o*A'	2-7 (*RAA*)

(6): 1- A*o*B (o → o) (contraposition)

∴ B'*o*A'

2- not [B'*o*A']	AP
3- B'*a*A'	2 (QN)
4- A*i*B'	1 (Ob)
5- B'*e*A	3 (Ob)
6- A*o*A	4, 5 (*F*)
7- B'*o*A'	2-6 (*RAA*)

(7): 1- B*a*C (***Barbara***)

2- A*a*B

∴ A*a*C

3- not [A*a*C]	AP
4- A*o*C	3 (QN)
5- A*i*C'	4 (Ob)
6- not [C*i*A]	AP
7- C'*e*A	6 (QN)
8- A*o*A	5, 7 (*F*)
9- C'*i*A	6-8 (*RAA*)
10- A*e*B'	2 (Ob)
11- C'*o*B'	9, 10 (*F*)
12- C'*i*B	11 (QN)
13- B*e*C'	1 (QN)
14- C'*o*C	12, 13 (*F*)
15- A*a*C	3-14 (*RAA*)

(8): 1- B*e*C (***Celarent***)

2- A*a*B

∴ A*e*C

3- not [A*e*C]	AP
4- A*i*C	3 (QN)
5- not [C*i*A]	AP
6- C*e*A	5 (QN)
7- A*o*A	4, 6 (*F*)
8- C*i*A	5-7 (*RAA*)
9- A*e*B'	2 (Ob)
10- C*o*B'	8, 9 (*F*)
11- C*i*B	10 (Ob)
12- C*o*C	11, 1 (*F*)
13- A*e*C	3-12 (*RAA*)

(9): 1- B*a*C (***Darii***)

2- A*i*B

∴ A*i*C

3- B*e*C'	1 (Ob)
4- A*o*C'	2, 3 (*F*)
5- A*i*C	4 (Ob)

In what follows, the auther proves one of the most complex moods in Aristotelian categorical logic, i.e. "***Bocarodo***" based of ***Ferio***.

(10):	1- B*o*C	***(Bocardo)***
	2- B*a*A	
	∴ A*o*C	
	3- not [A*o*C]	AP
	4- A*a*C	3 (QN)
	5- not [B*a*C]	AP
	6- B*o*C	5 (QN)
	7- B*i*C'	6 (Ob)
	8- not [C'*i*B]	AP
	9- C'*e*B	8 (QN)
	10- B*o*B	7, 9 (*F*)
	11- C'*i*B	8-10(*RAA*)
	12- B*e*A'	2 (Ob)
	13- C*o*A'	11, 12 (*F*)
	14- C'*i*A	13 (Ob)
	15- A*e*C'	4 (Ob)
	16- C'*o*C'	14, 15 (*F*)
	17- B*a*C	5-16 (*RAA*)
	18- B*e*C'	17 (Ob)
	19- B*i*C'	1 (Ob)
	20- not [C'*e*B]	AP
	21- C'*i*B	20 (QN)
	22- C'*o*C'	21, 18 (*F*)
	23- C'*e*B	20-22 (*RAA*)
	24- B*o*B	19, 23 (*F*)
	25- A*o*C	3-24 (*RAA*)

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